Probabilistic Analysis and Simulation of Line-of-Sight dependent Dynamic Switching Networks

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Abstract

Networks based on optical communication have dynamic links. These links only exists when there is a close to Line-of-Sight (LOS)between the corresponding robot nodes. In this work we first formulated a scenario of multiple robots equipped with optical communication devices. Randomness is introduced in the rotational behavior of optical device to increase the chances of getting (LOS). For simulation purpose, we defined some protocols which can be practically realized to start and maintain the communication link. Simulation results shows the feasibility for convergence of agreement protocol in such a system.

1 Introduction

However old this maybe, underwater exploration is still a very popular field, with application to marine sciences, environmental engineering, and oil/gas exploration among others. Autonomous robots and vehicles are playing an increasingly important role in carrying out these missions. A key task these vehicles need to do is to communicate with each other or with the base station, during their underwater deployment. While one can use tethered communication channels, wireless links are desirable and sometimes necessary. Radio frequency signals are heavily attenuated in water [1]. Acoustic communication is currently the prevalent method but suffers from low data rates, high latency, and high power consumption [2].

Over the past few years, light-emitting diode (LED)-based optical communication has emerged as a promising low-power, high-rate, economical solution to underwater data transfer [3–5]. However, optical communication is not omni-directional in nature like wifi, radio-waves and other wavelet propagation based communication. It requires close to Line of Sight between transmitter and receiver to begin and maintain the communication. By Close-to-Line-of-Sight we mean that the receiver and transmitter are not exactly pointing towards each other but they are in each other's field of view/field of transmission. From now on, in the rest of this report we would be using the terms 'Close-to-Line-of-Sight' and 'Line-of-Sight'(LOS) interchangeably.

Now, consider a scenario of robot nodes in a network, each one of which is equipped with optical-communication-device. Now each of the robot is rotating its optical device randomly to search for LOS with other robot. When they get LOS, they try to establish communication link, after successful communication, they start looking for other nodes for communication. One can see that at any given instant of time there can only be a single communication link associated with a node if there is any. At any given point of time, there would be pair of nodes with link between them. Fig. 1 shows one instant of dynamic switching scenario between the nodes. For illustration purpose, two edges are shown in the network, which is the maximum number of edges possible for 5 node network. However the number of edges in the network need not to be always the maximum. There would be cases when there is no edge in the network.

The rest of the report is organized as follows. In Section 2 the model for the received light intensity is presented in terms of the transmitter/receiver orientations and the distance. Then simple criteria for maintaining communication is presented which is used in this work. In section 3 the protocol for starting and maintaining communication is discussed in detail. Moving forward in section 4, probabilistic framework in achieving LOS between two robots is discussed. Simulation results are presented in section 5. Finally, concluding remarks are provided in Section 6.

2 Light Signal Strength Model

Consider a transmitter and receiver scenario shown in Fig. 2 used in [6]. The variables for interest are transmission angle θ, transmission distance d, and the angle of incidence ϕ. The line joining receiver and transmitter is communication line. The angle between transmitter's normal and communication line is transmission angle. Receiver angle or angle of incidence

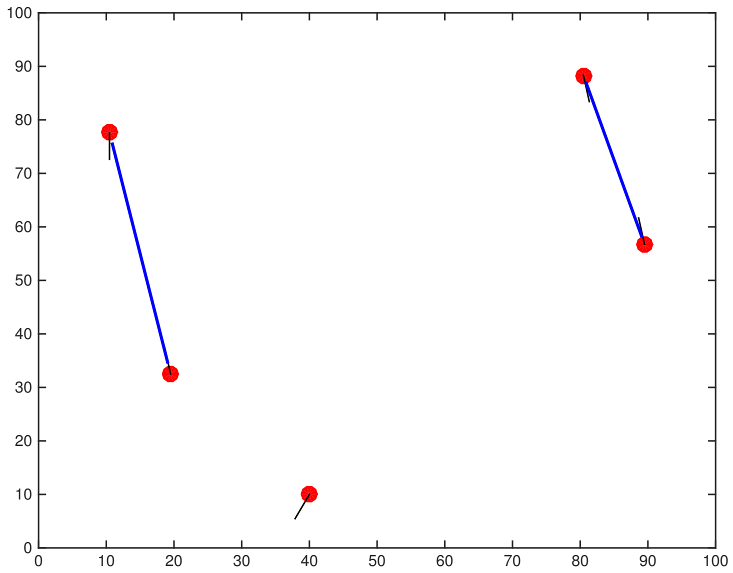


Figure 1: An illustration of the dynamic switching network

is the angle between receiver's normal and communication line. The transmission distance d is the distance of transmitter from the receiver.

Now, the analog voltage V_d corresponding to intensity of light received at receiver is modeled in Eq. 1. If V_d is more than some predefined threshold, which means they are in LOS, then one can say the receiver and transmitter are ready for communication.

$$F = -\frac{\sqrt{2\,t}\,\left(\mathrm{erf}\left(\left(\pi - \mathrm{x1}\right)\,\sqrt{-\frac{t}{2}}\right)\,\left(\pi - \mathrm{x1}\right) - \mathrm{erf}\left(\left(\pi + \mathrm{x1}\right)\,\sqrt{-\frac{t}{2}}\right)\,\left(\pi + \mathrm{x1}\right) + \frac{\sqrt{2}\,\mathrm{e}^{\frac{t\,\left(\pi + \mathrm{x1}\right)^2}{2}}\,\mathrm{i}}{\sqrt{\pi}\,\sqrt{t}} - \frac{\sqrt{2}\,\mathrm{e}^{\frac{t\,\left(\pi - \mathrm{x1}\right)^2}{2}}\,\mathrm{i}}{\sqrt{\pi}\,\sqrt{t}}\right)}{2\,\sqrt{\pi}\,\sqrt{-\frac{t}{2}}} \tag{1}$$

3 Assumptions and Communication Protocol

For ease of analysis and simplicity of computation in simulation we made some assumptions:

- All the robots are moving or standing on a plane: For the current work the robots are considered to be moving or standing on a plane but for later experiments in underwater settings this assumption would also needs to be relaxed.
- The robots are transparent, point light source: We introduced this assumption so that we don't need to worry about one robot coming in between two robots and blocking the communication link. In reality there is a possibility of this, but in a random scene the probability of three robots in a line is really low and can be ignored in basic analysis.

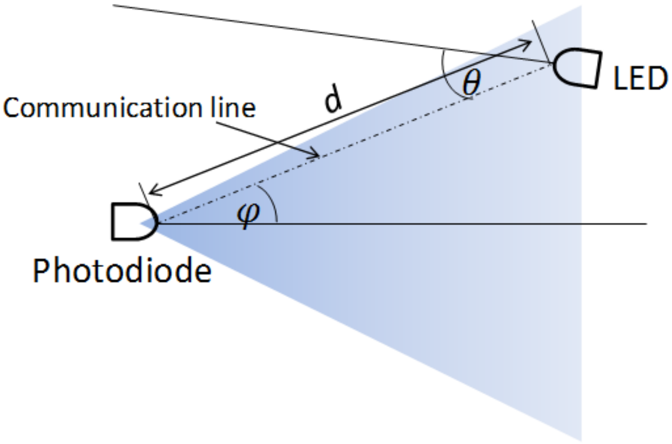


Figure 2: An illustration of the dynamic switching network

- If the transmission angle θ, transmission distance d and angle of incidence ϕ are inside certain predefined range then the receiver transmitter pair have a LOS. Ideally the intensity received V_d according to Eq. 1 defines whether the robots can communicate or not but for simplicity in computation in simulation this assumption is made.
- At a given time instant, a robot can interact with only one robot even if there are multiple options available: There is a chance that for a single robot two or more robots have LOS with it, however to have an interactive bidirectional communication with handshaking and confirmations one robot can only communicate with one robot at a time. The situation of LOS with multiple robots is handled in our protocol which is discussed next.

Consider the case depicted in Fig. 1. In the beginning, each robot have its optical device pointing in random direction which is distributed uniformly from -180^o to 180^o. Also they start rotating with an angular velocity ω which is chosen randomly from uniformly distribution $-\omega_{max}$ to ω_{max}. Where ω_{max} is the magnitude of the maximum allowable angular velocity. Assuming each robot has some allocated identity i which in our case is just a number. The robots starts transmitting its node number i as soon as it starts rotating. When a node comes in LOS with other node, it would receive other node number, say j. To initiate the communication from i's side it will start transmitting its own identity with other j robot's identity. In our case i robot will transmit $10j+i$. Now if robot i receives $10i+j$ back, it means j has also acknowledged i's presence and want to start the communication. So now when both robots receives their own identity(number) in initially received information. They start the communication and starts exchanging advanced information like their position and other states. Suppose a robot receives multiple node numbers at the same time, which means he has multiple options for communication then it chooses one of the option randomly and transmit that node number back with its own number. This way it still start communication with one node only, the other nodes who do not receive the confirmation back, starts rotating randomly again transmitting their node numbers. Once the communication is started between the robots they will communicate and update their states for a predefined period of time and then call their communication off. After the communication is over they will again choose new random ω to rotate. To avoid the possibility of interaction with the same node again they both will rotate 90^o quickly and then start transmitting their node number to search for communication options.

4 Brief Probabilistic Analysis

Consider two robots with initial optical device headings ϕ_1 and ϕ_2. This would correspond to a point on a 2D plane of ϕ_1 and ϕ_2. As shown in Fig. 3, different arrows denote the trajectory of relative angular orientation in 3 different cases. The shaded patch in the figure denotes the area where there is LOS. Consider the initial condition P_1 (blue arrow in Fig. 3) , with same angular velocities of robots. Since the arrow crosses the shaded region it means after sometime there would be LOS. However for initial condition P_2 (red arrow in Fig. 3), there would never be a LOS. For point P_3 (yellow arrow in Fig. 3), the angular velocities of robots is different, the arrow does not intersects the shaded region, however because of periodicity of angles, the arrow will encounter the copy of shaded region sometimes later and will land into the LOS.

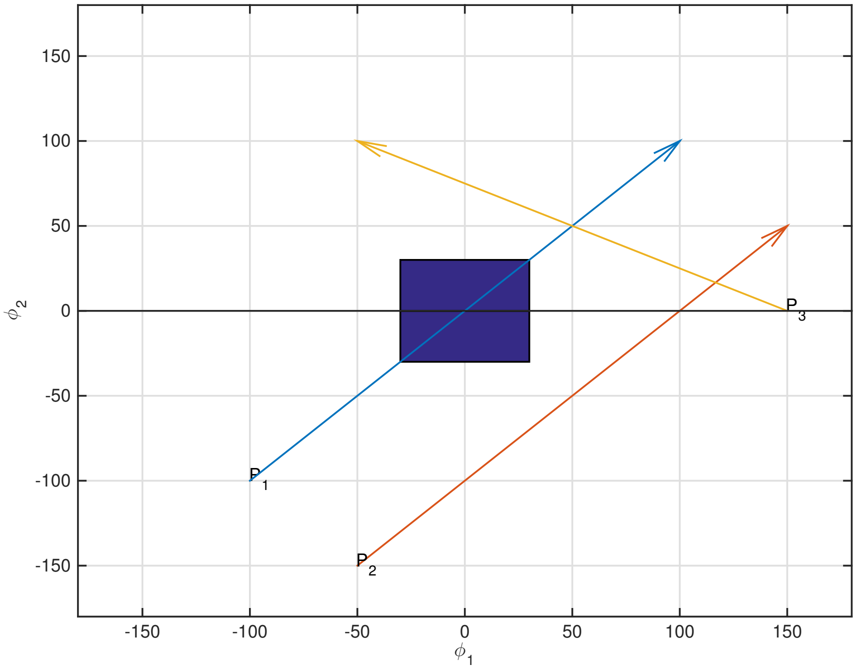


Figure 3: Trajectories of different relative angular orientations between two robots ϕ_1,ϕ_2 plane

Because of the periodicity of the LOS function over angles, this situation can be visualize on a Toroidal surface: Fig. 4.

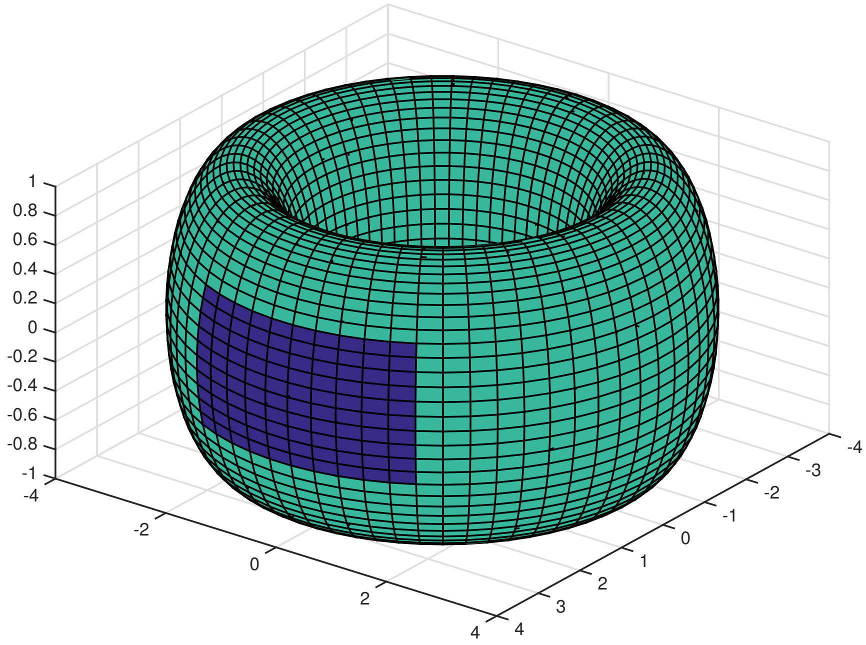
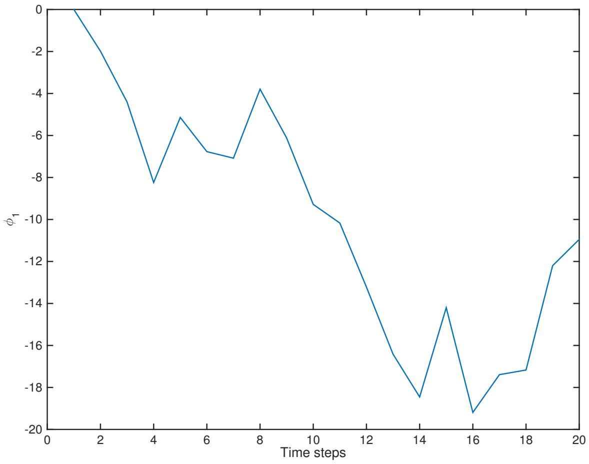


Figure 4: Since LOS function is periodic over orientation angles, ϕ_1,ϕ_2 plane can be wrapped to a Toroidal surface

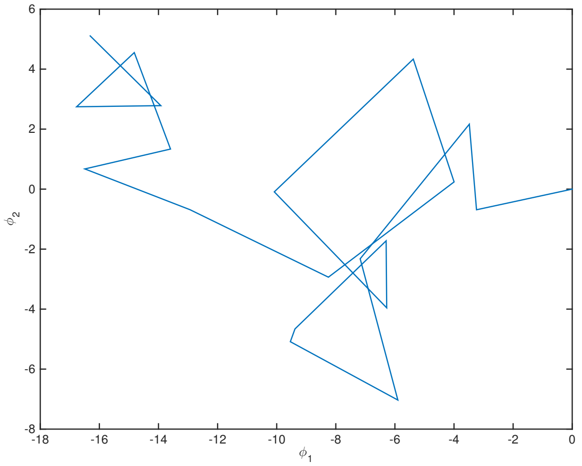
Now consider a different rotation strategy. Both robots starts with a randomly chosen angular velocity after rotating for time T they again choose their random velocities and repeat this process forever. Now consider the motion of ϕ_1 or ϕ_2 based on this strategy and the definition of Brownian Motion [7]: A stochastic process $\{X(t), t \geq 0\}$ is said to be a Brownian Motion process if

1. $X(0) = 0$; We can always shift the origin to satisfy this criteria
2. $\{X(t), t \geq 0\}$ has stationary and independent increments ; In our strategy also after each time interval of length T there is increment in $\phi_i(t)$. Since angular velocity ω is always chosen from the same uniform distribution, the increment is stationary. Also each time ω is chosen independent of previous values, the increments are also independent.
3. For every $t > 0$, $X(t)$ is normally distributed with mean 0 and variance $\sigma^2 t$; Here in this case the time of increment $\Delta t \rightarrow 0$ however in our case the limit is finite T, which is similar to random walk case from which Brownian motion is derived.

Now we can see that the motion is not exactly but can be approximated to Brownian motion and hence the results for Brownian motion can be used for analysis. From Fig. 5a and 5b , it can be seen that the angles are performing Brownian motion. Fig. 5a shows ϕ_1



(a) Discrete Brownian motion in one dimension: plot with $T = 1$



(b) Discrete Brownian motion in two dimension on ϕ_1 ϕ_2 plane

Figure 5: Angular positions of robots as Brownian motion

over time with T as 1 unit. Fig. 5b shows 2D Brownian motion trajectory of relative angular position on ϕ_1 ϕ_2 plane.

It can be shown that when a point performs a Brownian motion on a Toroidal surface, there is a positive probability that it will reach any given part of the surface in finite time. Which means if we use this strategy to rotate the optical device of the robots. There is always a chance of LOS between the robots.

5 Simulation Results

Based on the framework and protocol defined in previous sections, the dynamic switching network is simulated on MATLAB®. Five robots are placed randomly on a plane and then the discussed strategy is implemented for switching.When agreement protocol [8] was implemented on robots states the states converges to a rendezvous. Fig. 6 shows the convergence of states to a single value.

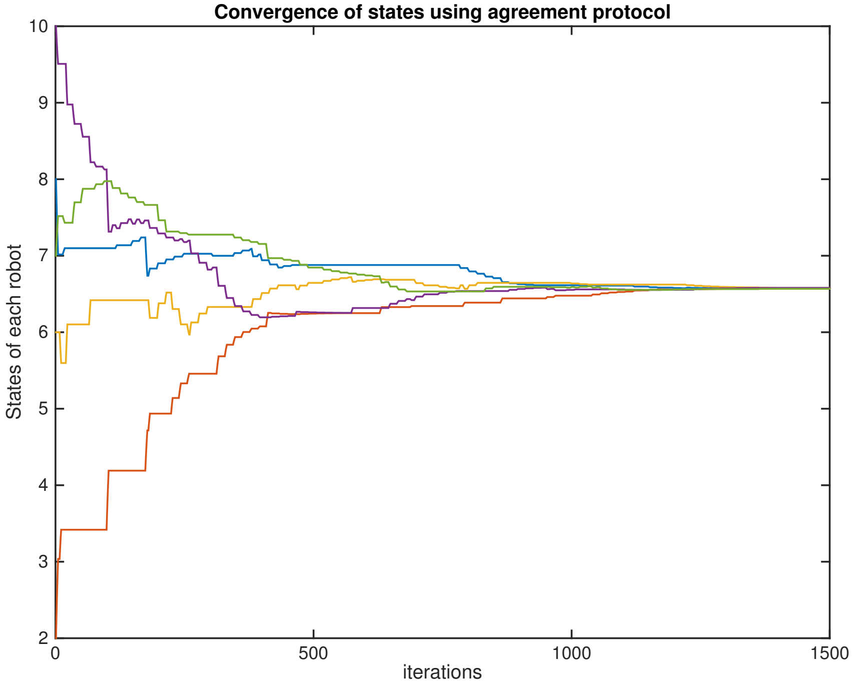


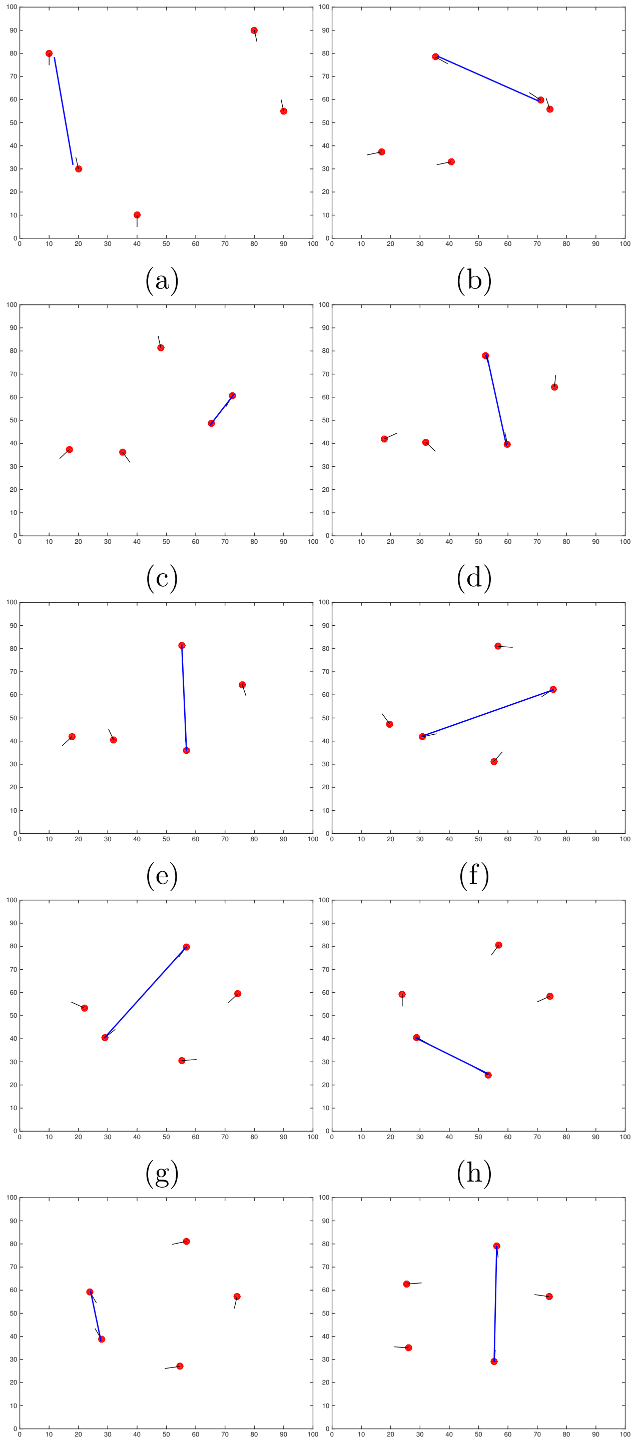
Figure 6: States of robots following agreement protocol converging to a single value

Fig. 7 shows the sequence of different time instances of a dynamic switching network when formation control is implemented. From Fig. 7i and Fig. 7j it can be observed that the nodes are able to maintain a pentagonal formation in the end.

One thing to be noted here that though the agreement protocol and formation control seems to be working well in this switching network, however it takes comparatively significantly larger number of steps to converge (1500 from Fig. 6) as oppose to less than 100 steps in static connected network.

6 Conclusion and Future Works

In this work dynamic switching behavior of a LOS based optical-communication network is explored. By introduction of randomness the rotational pattern of optical devices, probabilistic framework is formulated, which is similar to Brownian motion. Based on the predefined protocol and strategy, the network is simulated. Simulation results shows the efficacy of



(a) (b)

(c) (d)

(e) (f)

(g) (h)

(i) (j)

Figure 7: Image sequence of network converging to pentagonal formation

agreement protocol over such dynamic network. Future work includes rigorous probabilistic analysis of the switching, which includes finding expected time for first LOS, Probabilistic analysis of multiple nodes and to relate this analysis with the rate of convergence of agreement protocol on the network.

7 References

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